ELECTROMAGNETIC SCATTERING FROM HETEROGENEOUS MEDIA
A comparison between numerical computations and experimental results

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ABSTRACT

In a view of its possible use for control of absorption, Radar Cross Section (RCS) and shielding, heterogeneous material coatings have received considerable attention recently. An analytical approach to the investigation of the scattering of electromagnetic waves by heterogeneous materials made of spherical inclusions is presented. The theory takes into account the coupling between the inclusion. Computed and experimental results in the range 1 to 100 GHz are given.

INTRODUCTION

In a view of its possible use for control of absorption, Radar Cross Section (RCS) and shielding, heterogeneous material coatings have received considerable attention recently. The numerous parameters arising in their development such as nature, shape, size or concentration of the constituents, makes necessary a previous modeling. The classical mixture laws usually used to modelize the electromagnetic behaviour of such materials generally fail when the inclusions sizes are in the same order of wavelength. This effect can occur easily in the millimetric frequency range. Then it is necessary to take into account couplings between the particles which leads to multiple scattering effects.
An analytical approach to the investigation of the scattering of electromagnetic waves by heterogeneous materials made of spherical inclusions is presented. It involves to solve the general wave propagation equation by using the Mie formalism (incident and scattered fields are developed on bases of vectorial spherical harmonics)[2,3]. At each step of frequency, the scattered field is deduced by using an iterative method and the T-matrix formalism which describes completely the properties of the inclusions [1]. A solution of this scattering problem in the case of 1D, 2D or 3D systems of spheres is presented to illustrate the theory. Computed results are compared with experimental results obtained by using vectorial network analyzers and free-space measurement technique.

ANALYSIS

Scattering by a sphere

A spherical particle of radius a is placed in a homogeneous medium lit by an electromagnetic plane wave of wavelength λ. The incident wave is scattered in conditions which depend on the electromagnetic characteristics (ε, μ) of the sphere. The incident and scattered fields will spread in the medium following Maxwell's equations that we can combine to form Helmoltz's wave equation. The solution of the scalar Helmoltz's equation, obtained by separation of variables, is a combination of Bessel's or Hankel's spherical functions, associated Legendre's and trigonometric functions. From this scalar equation, we can build a vectorial base of vectors of spherical harmonics $(\vec{M}_{mn}^{(i)}, \vec{N}_{mn}^{(i)})$ with i = 1 or 3. i = 1 is used in the expression of the incident field: Bessel's functions express the regularity at the origin, while when i = 3, Bessel's functions are replaced by Hankel's functions to express the convergence on the infinite of the scattered field. Thus, the incident field is written:

$$\vec{E}^i(r,\theta,\varphi) = \sum_{n=0}^{\infty}\sum_{m=-n}^{n}\Big[q_{mn}(\theta_k,\varphi_k)\,\vec{M}_{mn}^{(1)}(r,\theta,\varphi) + p_{mn}(\theta_k,\varphi_k)\vec{N}_{mn}^{(1)}(r,\theta,\varphi)\Big]$$

$$\vec{H}^i(r,\theta,\varphi) = \frac{-j}{Z}\sum_{n=0}^{\infty}\sum_{m=-n}^{n}\Big[p_{mn}(\theta_k,\varphi_k)\,\vec{M}_{mn}^{(1)}(r,\theta,\varphi) + q_{mn}(\theta_k,\varphi_k)\vec{N}_{mn}^{(1)}(r,\theta,\varphi)\Big]$$

(θ_k, φ_k) being the incident angles of the wave vector and Z the impedance of the host medium.

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The scattered field coefficients can be deduced from the incident ones through the T-matrix which is calculated from the boundary conditions on the surface of the particle.

$$\begin{bmatrix} P_{mn}(a,\varepsilon,\mu,\theta_k,\varphi_k) \\ Q_{mn}(a,\varepsilon,\mu,\theta_k,\varphi_k) \end{bmatrix} = T(a,\varepsilon,\mu) * \begin{bmatrix} p_{mn}(\theta_k,\varphi_k) \\ q_{mn}(\theta_k,\varphi_k) \end{bmatrix}$$

This matrix contains all the informations about the nature of the particle : its size, shape, and electromagnetic properties.

Multiple scattering

If there are N spheres in the medium, each of them scatters the incident electromagnetic wave. Coupling effects can appear and each sphere will be lit by an incident wave very different from the initial incident plane wave. At whatever point in the medium, the total field is the sum of the incident field and of the fields scattered by all the particles.
The method used to calculate the total scattered field is the iterative method developed by Hamid and Ciric [4] :
Whatever the number and the disposition of the spheres, we calculate, in the first time, the field scattered by each sphere as if it was alone in the incident field. This scattered field is called the first-order scattered field. This first-order scattered field becomes incident on the *n-1* other spheres. Each sphere scatters this incident field. This scattered field is called the second-order scattered field and so on. The total scattered field is the sum of all the contributions at each order. This iterative process continues until the solution converges. We need to calculate the translational coefficients in order to express the scattered field by one sphere in terms of incident field for the other spheres. From a computational viewpoint, the calculation of these translational coefficients requires the most part of the time.
We take place in a referential $\mathcal{R}$ connected to the center of a sphere. The first-order scattered field can be written :

$$\begin{pmatrix} \vec{E}_1^s(r,\theta,\varphi) \\ \vec{H}_1^s(r,\theta,\varphi) \end{pmatrix} = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \begin{bmatrix} Q_1^{\Re}(n,m) & P_1^{\Re}(n,m) \\ P_1^{\Re}(n,m) & Q_1^{\Re}(n,m) \end{bmatrix} \cdot \begin{pmatrix} \vec{M}_{mn}^{(3)}(r,\theta,\varphi) \\ \vec{N}_{mn}^{(3)}(r,\theta,\varphi) \end{pmatrix}$$

At the second iteration, this first-order scattered field is incident on the neighbouring spheres.

By using the next translation equations between $\mathcal{R}$ and $\mathcal{R}'$:

$$\begin{pmatrix} \vec{M}_{mn}^{(3)}(r,\theta,\varphi) \\ \vec{N}_{mn}^{(3)}(r,\theta,\varphi) \end{pmatrix} = \sum_{\nu=0}^{\infty} \sum_{\mu=-\nu}^{\nu} \begin{bmatrix} A_{\mu\nu}^{mn}(\Re,\Re') & B_{\mu\nu}^{mn}(\Re,\Re') \\ B_{\mu\nu}^{mn}(\Re,\Re') & A_{\mu\nu}^{mn}(\Re,\Re') \end{bmatrix} \cdot \begin{pmatrix} \vec{M}_{\mu\nu}^{(1)}(R,\Theta,\Phi) \\ \vec{N}_{\mu\nu}^{(1)}(R,\Theta,\Phi) \end{pmatrix}$$

we obtain the expressions of the new incident fields.
The translation coefficients have been developed by Orval R.Cruzan and al [5]. To compute the higher order scattered fields coefficients, we use recursive relations.

The Radar Cross Section (R.C.S) for a three dimensional scattering problem is defined as :

$$\sigma(\theta,\phi) = \lim_{R\to\infty}\left(4\pi R^2 \left|\frac{\vec{E}^s}{\vec{E}^{inc}}\right|^2\right)$$

We assume that the incident wave is a unit plane wave. By using the expansion of the incident field and scattered fields on the MIE base, we obtain :

$$\sigma(\theta,\phi) = \frac{4\pi}{k^2}\left(\left|A_{mn}(\theta,\phi)\right|^2 + \left|B_{mn}(\theta,\phi)\right|^2\right)$$

where

$$A_{mn}(\theta,\phi) = \sum_{n=0}^{\infty}(-i)^n\left(\sum_{m=-n}^{n}\left[Q_{mn}\frac{m}{\sin\theta}P_n^m(\cos\theta) + P_{mn}\frac{\partial}{\partial\theta}P_n^m(\cos\theta)\right]e^{im\phi}\right)$$

$$B_{mn}(\theta,\phi) = \sum_{n=0}^{\infty}(-i)^{n+1}\left(\sum_{m=-n}^{n}\left[Q_{mn}\frac{\partial}{\partial\theta}P_n^m(\cos\theta) + P_{mn}\frac{m}{\sin\theta}P_n^m(\cos\theta)\right]e^{im\phi}\right)$$

COMPUTATION DATA and EXPERIMENTAL RESULTS

At each step of the calculation, numerical computations were checked experimentaly by using a free space measurement method. A bistatic system is driven by two step motors to have a good accuracy in the position of the emitting and receiving antennas. Measurements are performed by using vectorial network analyzers ABmm (15-100 GHz) and HP 8510B (8-18 GHz).

In order to check the computation data, we made comparisons with lot of configurations. For instance, the results for three (figure 1a) and four spheres (figure 1b) are given. As we can observe, there is a good agreement between the simulation and the measurements. The small shift between the resonant frequencies is due to the elongation of the wavelength in a focussed beam produced by an antenna with a circular aperture [6].

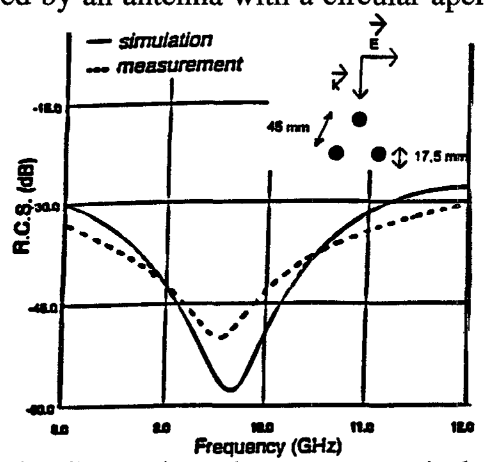

Figure 1a: Comparisons between numerical and experimental results for three perfectly conducting spheres.

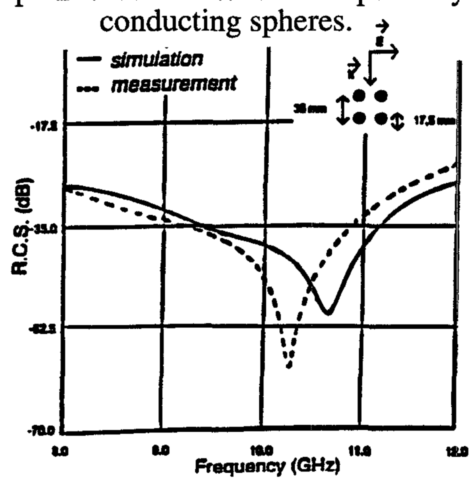

Figure 1b: Comparisons between numerical and experimental results for four perfectly conducting spheres.

SCATTERING DIAGRAM

As can be observed in figure 2a, 2b and 2c, coupling and resonant effects cannot be neglected.

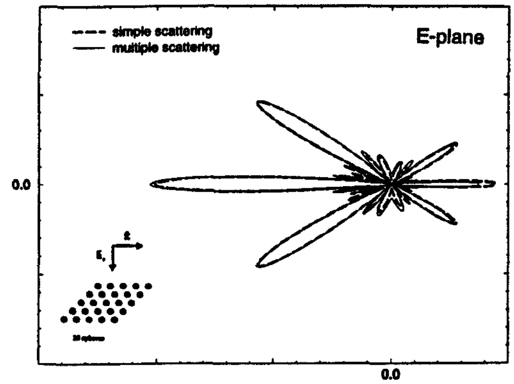

Figure 2a: Scattering diagram for a configuration of 25 spheres. Coupling and resonant effects in the E-plane.

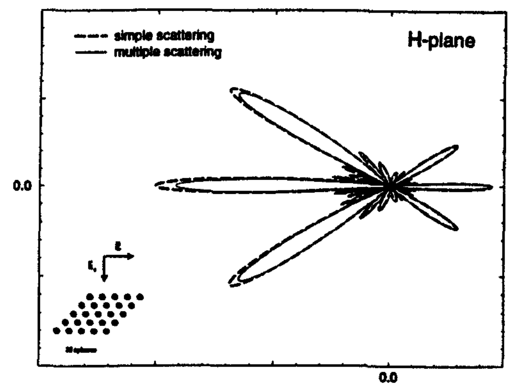

Figure 2b: Scattering diagram for a configuration of 25 spheres. Coupling and resonant effects in the H-plane.

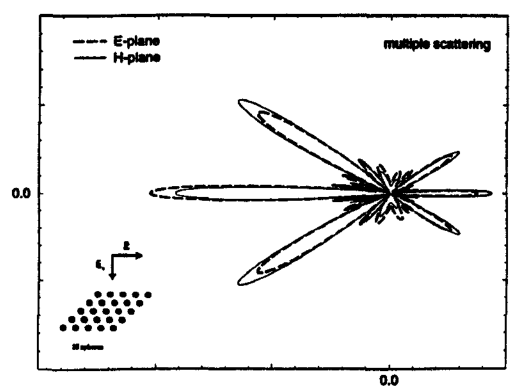

Figure 2c: Scattering diagram for a configuration of 25 spheres. Differences between the E-plane and the H-plane.

The figure 3 underscores some multiple scattering effects inside a 3D structure and confirm the fact that the main interactions in the E-plane are given by an additional layer of spheres perpendicular to the E-plane.

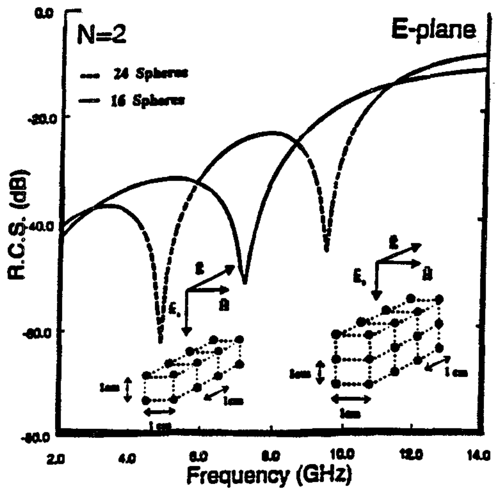

Figure 3 : Influence of an additional layer of spheres in a plane perpendicular to the E-plane. a = 0.3 cm

CONCLUSION

A solution of electromagnetic scattering from a 3D network of spheres by using an iterative method is given. This method allows to take into account the coupling and the resonant effects that can appear in some configurations. This model can easily be used with various nature (dielectric stratified or chiral ones), with various shape (cylinders, ...) and allows to calculate as well the far-field as the near-field.

References

1. Y.C. Tzeng, A.K. Fung, 1994, "T-matrix Approach to Multiple Scattering of EM Waves from N-spheres."
JEWA, vol 8 n°1, 61-84

2. G. Mie, 1908, "Beiträge zur Optik trüber Medien, speziell kolloidaler Metallösungen."
Ann. Phy., vol 25, 377

3. J.A. Stratton, 1941, "Electromagnetic Theory"
McGraw-Hill, New-York, USA

4. A.K. Hamid, I.R. Ciric, M. Hamid, Dec. 1991, "Iterative solution of the scattering by an arbitrary configuration of conducting or dielectric spheres."
IEE Proceedings-H, vol 138 n°6

5. O.R. Cruzan, 1962, "Translational addition theorems for spherical vector wave functions"
Quat. Appl. Math., vol 20, n°1, 33-40

6. J. Musil, F.Zacek, 1982, "Microwave measurements of complex permittivity by free space methods and their applications", 45-60
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